Tidewater Big Data Enthusiasts: Exploring Shuffling Cards

Chuck Cartledge, PhD

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1 Introduction

"When in the course of human events ..." Nope wrong introduction. Let's try again.

We are interested in the mechanics of shuffling cards. In many introduction to statistics classes, the number of possible permutations of a deck of 52 cards is 52! or approximately $8.0658175 * 10^{67}$. Far too many to deal with (pun intended), so we turned our attention to a more manageable problem: How many times must a 52 card deck be perfectly shuffled in order to return all cards to their original position in the deck?

We will write an R script to answer the initial question, expand the script to address imperfect shuffling, and expand it again to see if decks of different sizes can also be returned to their original state.

2 Background

We were reading a book about HUGE numbers, and came across this:

> *"... A prized magician's skill is that of a perfect shuffle – splitting the deck exactly in half and then interleaving the two sets in a precisely alternating pattern while keeping the same card on top each time. Far from randomizing the deck, it takes just eight repetitions to return the cards to exactly their original order. ..."*
>
> R. Elwes [1]

On the surface, this claim seemed counter intuitive and easily proved or disproved via a simulation. So, we wrote an R script that performed a perfect riffle shuffle, specifically an "out shuffle."

Here is a short list of different types of shuffles[2]:

- **52 pickup** A person may throw a deck of cards into the air or across a surface, and then pick up the cards in random order, assembled with the cards facing the same direction.
- **Box** The deck is held from the top by one hand close to the top of the table, and a pile is stripped off the top of the deck with the other hand and placed on the table.
- **Corgi** Also known as the Chemmy, Irish, wash, scramble, hard shuffle, ice cream, smooshing, or washing the cards, this involves simply spreading the cards out face down, and sliding them around and over each other with one's hands.
- **Cut** Typically performed after a previous shuffling method, the cut is simply taking a deck, dividing it into two portions of random size, and putting the previously lower portion on top of the previously higher portion.

- **Faro** Weaving is the procedure of pushing the ends of two halves of a deck against each other in such a way that they naturally intertwine. Sometimes the deck is split into equal halves of 26 cards which are then pushed together in a certain way so as to make them perfectly interweave.
- **Hindu** The deck is held face down, with the middle finger on one long edge and the thumb on the other on the bottom half of the deck. The other hand draws off a packet from the top of the deck. This packet is allowed to drop into the palm. The maneuver is repeated over and over.
- **Mexican spiral** The Mexican spiral shuffle is performed by cyclic actions of moving the top card onto the table, then the new top card under the deck, the next onto the table, next under the deck, and so on until the last card is dealt onto the table.
- **Mongean** The Mongean shuffle, or Monge's shuffle, is performed as follows (by a right-handed person): Start with the unshuffled deck in the left hand and transfer the top card to the right. Then repeatedly take the top card from the left hand and transfer it to the right, putting the second card at the top of the new deck, the third at the bottom, the fourth at the top, the fifth at the bottom, etc.
- **Pile** Cards are simply dealt out into a number of piles, then the piles are stacked on top of each other.
- **Riffle** In which half of the deck is held in each hand with the thumbs inward, then cards are released by the thumbs so that they fall to the table interleaved.
- **Team Shuffle** Especially useful for large decks, a shuffler may divide a deck into two or more smaller decks, and give the other portion(s) to (an)other shuffler(s), each to choose their own shuffling method(s).

3 Results

The inspirational quote addressed a perfect out shuffle riffle. Here we expand on that idea.

3.1 A 52 card deck

A perfect out shuffle riffle takes one card from the top of the left half of the deck, then and interleaves it with the one card from the top of the right half of the deck. An R script was written to perform a perfect riffle shuffle (see Section 6), and data was collected (see Table 1). In a perfect out shuffle riffle, the top and bottom cards retain their location, and that is seen in the table.

Orig	Shuffle 1	Shuffle 2	Shuffle 3	Shuffle 4	Shuffle 5	Shuffle 6	Shuffle 7	Shuffle 8
1	1	1	1	1	1	1	1	1
2	27	14	33	17	9	5	3	2
3	2	27	14	33	17	9	5	3
4	28	40	46	49	25	13	7	4
5	3	2	27	14	33	17	9	5
6	29	15	8	30	41	21	11	6
7	4	28	40	46	49	25	13	7
8	30	41	21	11	6	29	15	8
9	5	3	2	27	14	33	17	9
10	31	16	34	43	22	37	19	10
11	6	29	15	8	30	41	21	11
12	32	42	47	24	38	45	23	**12**
13	7	4	28	40	46	49	25	13
14	33	17	9	5	3	2	27	14
15	8	30	41	21	11	6	29	15
16	34	43	22	37	19	10	31	16
17	9	5	3	2	27	14	33	17
18	35	18	35	18	35	18	35	18
19	10	31	16	34	43	22	37	19
20	36	44	48	50	51	26	39	20
21	11	6	29	15	8	30	41	21
22	37	19	10	31	16	34	43	22
23	**12**	32	42	47	24	38	45	23
24	38	45	23	**12**	32	42	47	24
25	13	7	4	28	40	46	49	25
26	39	20	36	44	48	50	51	26
27	14	33	17	9	5	3	2	27
28	40	46	49	25	13	7	4	28
29	15	8	30	41	21	11	6	29
30	41	21	11	6	29	15	8	30
31	16	34	43	22	37	19	10	31
32	42	47	24	38	45	23	**12**	32
33	17	9	5	3	2	27	14	33
34	43	22	37	19	10	31	16	34
35	18	35	18	35	18	35	18	35
36	44	48	50	51	26	39	20	36
37	19	10	31	16	34	43	22	37
38	45	23	**12**	32	42	47	24	38
39	20	36	44	48	50	51	26	39
40	46	49	25	13	7	4	28	40
41	21	11	6	29	15	8	30	41
42	47	24	38	45	23	**12**	32	42
43	22	37	19	10	31	16	34	43
44	48	50	51	26	39	20	36	44
45	23	**12**	32	42	47	24	38	45
46	49	25	13	7	4	28	40	46
47	24	38	45	23	**12**	32	42	47
48	50	51	26	39	20	36	44	48
49	25	13	7	4	28	40	46	49
50	51	26	39	20	36	44	48	50
51	26	39	20	36	44	48	50	51
52	52	52	52	52	52	52	52	52

Table 1: Results of a perfect out shuffle riffle shuffle of a 52 card deck. The highlighted cell show how the 8^{th} card moves through the deck. It ultimately returns to its original place.

An imperfect riffle may take more than one card from each half of the deck in interleave those "blocks" of cards. The perfect riffle R script was modified to support imperfect divisions of the deck to see if the cards eventually return to their original positions. Imperfect values of seven cards for the left hand half and 15 for the right hand half were captured (see Table 2), and the cards do eventually return to their original location. Those particular values for left and right hand halves were selected for tabulation based on these criteria:

- The tracked location (the 8^{th} card) wasn't too large, and
- The number of shuffles to return the cards was small (10 in this case).

The selection of these values was determined by simulating all possible imperfect left and right hand values, and reporting one combination that met the tabulation criteria.

There are 2,704 (52 * 52) possible combinations of imperfect cards taken from the left and right hand divisions of a 52 card deck. It is impractical to print out and evaluate all combinations as a set of table. Hence, the R script was modified to process all possible divisions and create a contour map of the number shuffles per right and left hand divisions it would take to return the deck to its original configuration (see Figure 1). A zoomed in map of the lower left area shows greater detail and tighter contour lines (see Figure 2).

A contour plot can show how the different values of left and right hand affect the number of shuffles until restoration. A histogram of the number of shuffles shows the number overall pattern of shuffles (see Figure 3).

The take away from these tables and images is that a 52 card deck can be returned to its original condition in 1 to 11,592 shuffles depending on how many cards are consistently taken from the right and left halves of the deck.

3.2 Different sized card decks

The inspirational quote was about a 52 card deck and perfect out shuffle riffle , but what about decks that aren't 52 cards? Is it possible to for a perfect riffle out shuffle to restore those decks to their original state? And if so, how many shuffles would it take per each deck size? The short answers are: Yes, and Yes (see Figure 4).

4 Future work

After looking at and playing with the data, a few areas come to mind:

- Casual examination of the tabulated data (see Table 1 and Table 2) hints at some sort of additive and modulo operation controlling where the cards (in those cases the 8^{th}) go.
- Looking at the number of shuffles needed to restore a deck (see Figure 1), there seem to be hints in the visualizations.

Orig	Shuffle 1	Shuffle 2	Shuffle 3	Shuffle 4	Shuffle 5	Shuffle 6	Shuffle 7	Shuffle 8	Shuffle 9	Shuffle 10
1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7	7	7
8	18	28	13	23	**8**	18	28	13	23	**8**
9	19	29	14	24	9	19	29	14	24	9
10	20	30	33	36	39	42	45	15	25	10
11	21	31	34	37	40	43	46	16	26	11
12	22	32	35	38	41	44	47	17	27	12
13	23	**8**	18	28	13	23	**8**	18	28	13
14	24	9	19	29	14	24	9	19	29	14
15	25	10	20	30	33	36	39	42	45	15
16	26	11	21	31	34	37	40	43	46	16
17	27	12	22	32	35	38	41	44	47	17
18	28	13	23	**8**	18	28	13	23	**8**	18
19	29	14	24	9	19	29	14	24	9	19
20	30	33	36	39	42	45	15	25	10	20
21	31	34	37	40	43	46	16	26	11	21
22	32	35	38	41	44	47	17	27	12	22
23	**8**	18	28	13	23	**8**	18	28	13	23
24	9	19	29	14	24	9	19	29	14	24
25	10	20	30	33	36	39	42	45	15	25
26	11	21	31	34	37	40	43	46	16	26
27	12	22	32	35	38	41	44	47	17	27
28	13	23	**8**	18	28	13	23	**8**	18	28
29	14	24	9	19	29	14	24	9	19	29
30	33	36	39	42	45	15	25	10	20	30
31	34	37	40	43	46	16	26	11	21	31
32	35	38	41	44	47	17	27	12	22	32
33	36	39	42	45	15	25	10	20	30	33
34	37	40	43	46	16	26	11	21	31	34
35	38	41	44	47	17	27	12	22	32	35
36	39	42	45	15	25	10	20	30	33	36
37	40	43	46	16	26	11	21	31	34	37
38	41	44	47	17	27	12	22	32	35	38
39	42	45	15	25	10	20	30	33	36	39
40	43	46	16	26	11	21	31	34	37	40
41	44	47	17	27	12	22	32	35	38	41
42	45	15	25	10	20	30	33	36	39	42
43	46	16	26	11	21	31	34	37	40	43
44	47	17	27	12	22	32	35	38	41	44
45	15	25	10	20	30	33	36	39	42	45
46	16	26	11	21	31	34	37	40	43	46
47	17	27	12	22	32	35	38	41	44	47
48	48	48	48	48	48	48	48	48	48	48
49	49	49	49	49	49	49	49	49	49	49
50	50	50	50	50	50	50	50	50	50	50
51	51	51	51	51	51	51	51	51	51	51
52	52	52	52	52	52	52	52	52	52	52

Table 2: Results of an imperfect out shuffle riffle shuffle of a 52 card deck. The highlighted cell show how the 8^{th} card moves through the deck. It ultimately returns to its original place.

5 Conclusion

Our explorations into how many shuffles it takes to restore a deck of cards to its original state started with the claim that it takes only eight perfect out shuffles riffle. The claim seemed too simple and we wrote an R script program to test the theory. The theory held true for a perfect out shuffle riffle shuffle. Even imperfect shuffles held true, although in some cases, it took 11,592 shuffles.

We then looked at decks of different sizes (from 10 to 1,024) to see if they also could be returned to their original state with a finite number of shuffles. They could.

There may be some sort of closed form equation to predict where a card could move to in a deck, but I'm tired and don't feel like pursuing that search right now.

6 Embedded files

Embedded files may not be extractable using conventional web browsers. They are extractable using Adobe Acrobat and Reader software by right-clicking on the paperclip and saving the file to a local directory.

Other techniques to extract the images and attachments from this file are:

- `pdfimages -png ./shuffle.pdf /tmp/Temp/` – will extract all the images in `shuffle.pdf` in the current directory to the `/tmp/Temp` directory as png image files.
- `pdftk shuffle.pdf unpack_files` – will extract all the attachments in `shuffle.pdf` to the current directory

The embedded scripts and files were written and tested using a specific file and directory structure (see Figure 5).

- shuffle05.R – The R script file used to simulate and visualize different out shuffle riffle shuffles.
- library.R – A collection of R functions used by different script iterations.

References

[1] Richard Elwes, *Huge numbers: A story of counting ambitiously, from 4 1/2 to fish 7*, Hachette UK, 2026.

[2] Wikipedia Staff, *Shuffling*, `https://en.wikipedia.org/wiki/Shuffling`, 2026.

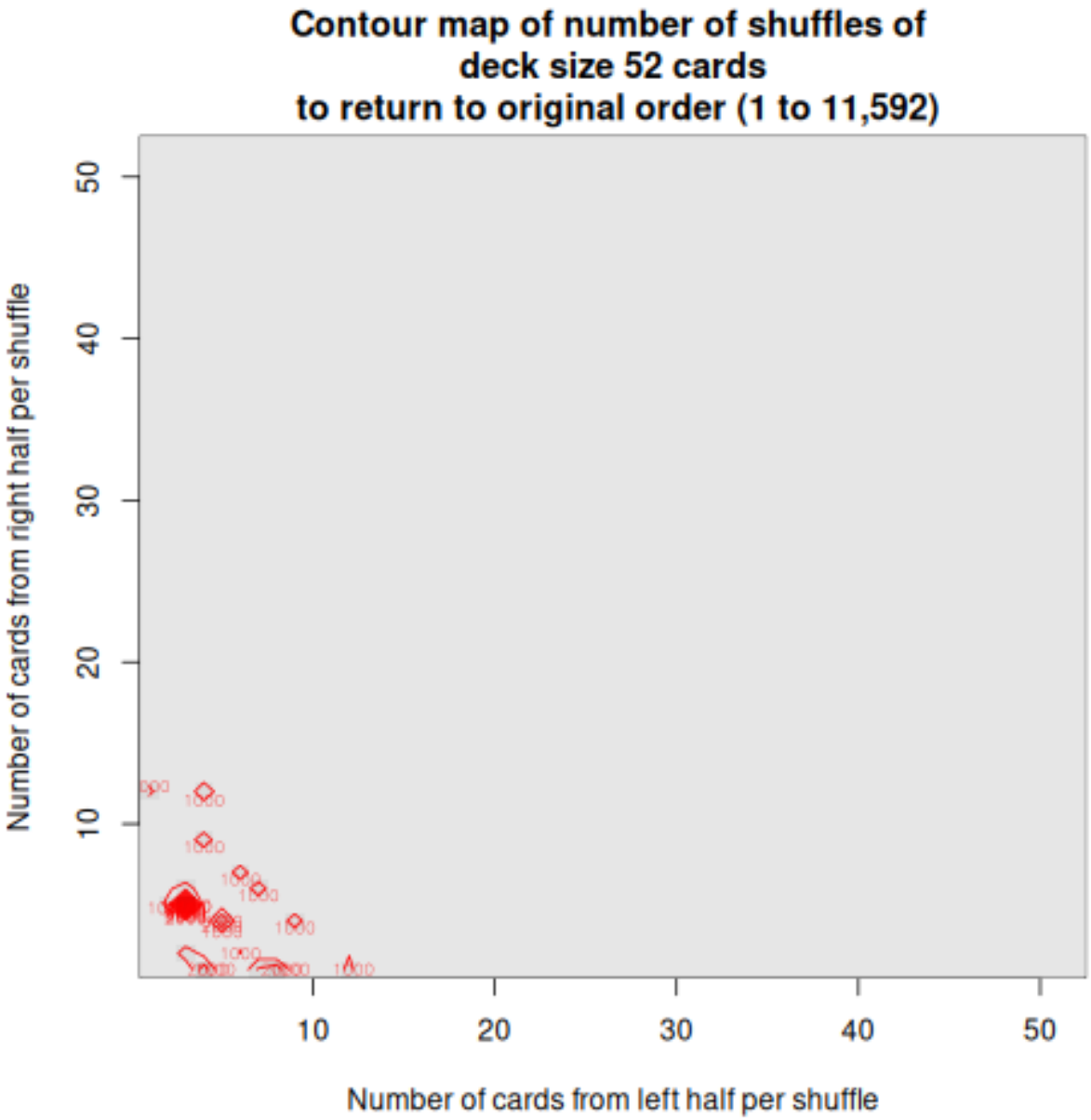

Figure 1: Contour map of all possible out shuffle riffle perfect combinations. Most of the notable activity is in the lower left corner, where the red contour lines are spaced at 1,000 shuffle increments. The vast majority of the plane has a height of about 1, meaning that the deck is returned to its original condition after one shuffle.

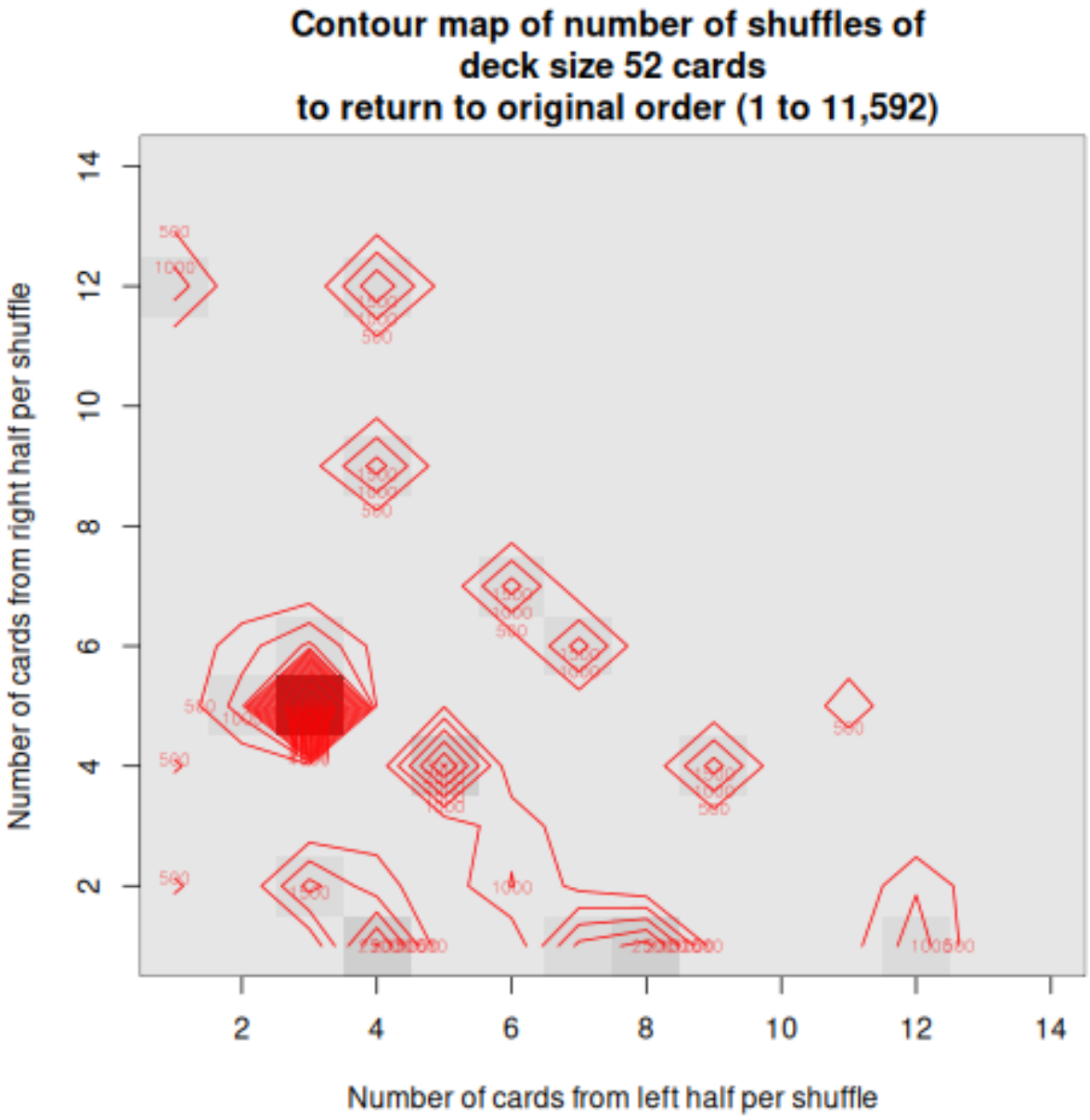

Figure 2: Zoomed contour map of all possible out shuffle riffle perfect combinations. A zoomed in portion of the overall contour map. Contour lines are now 500 shuffles apart. The highest peak is 11,592 when 3 cards are taken from the left hand and 5 are taken from the right.

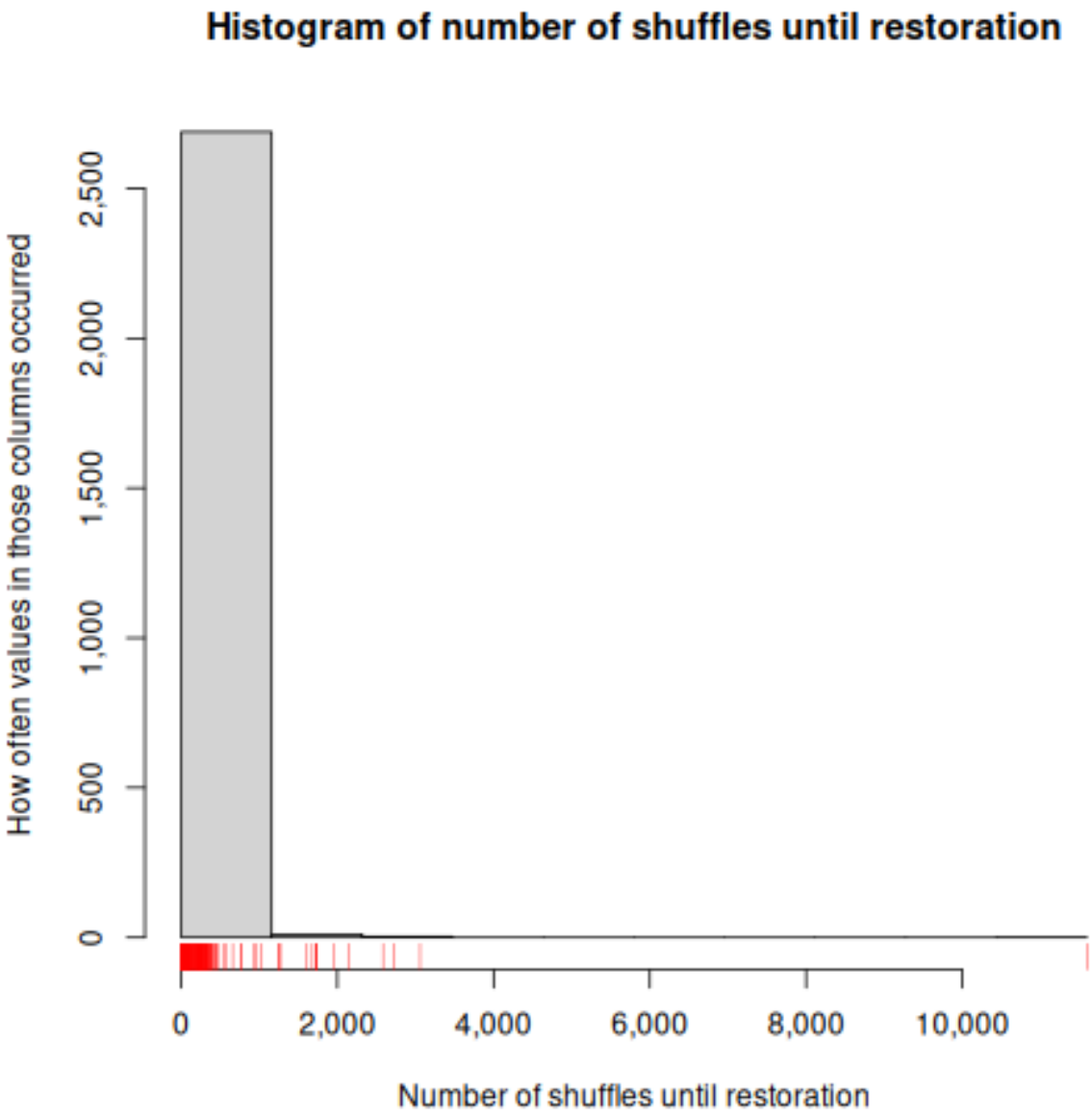

Figure 3: Histogram of number of shuffles to restore a 52 card deck. The vertical red lines under the columns approximates the number of times a value appeared in that histogram bin. The vast majority (2,689 out of 2,704 or $> 99\%$) occur in the first bin from 1 to 1,160.

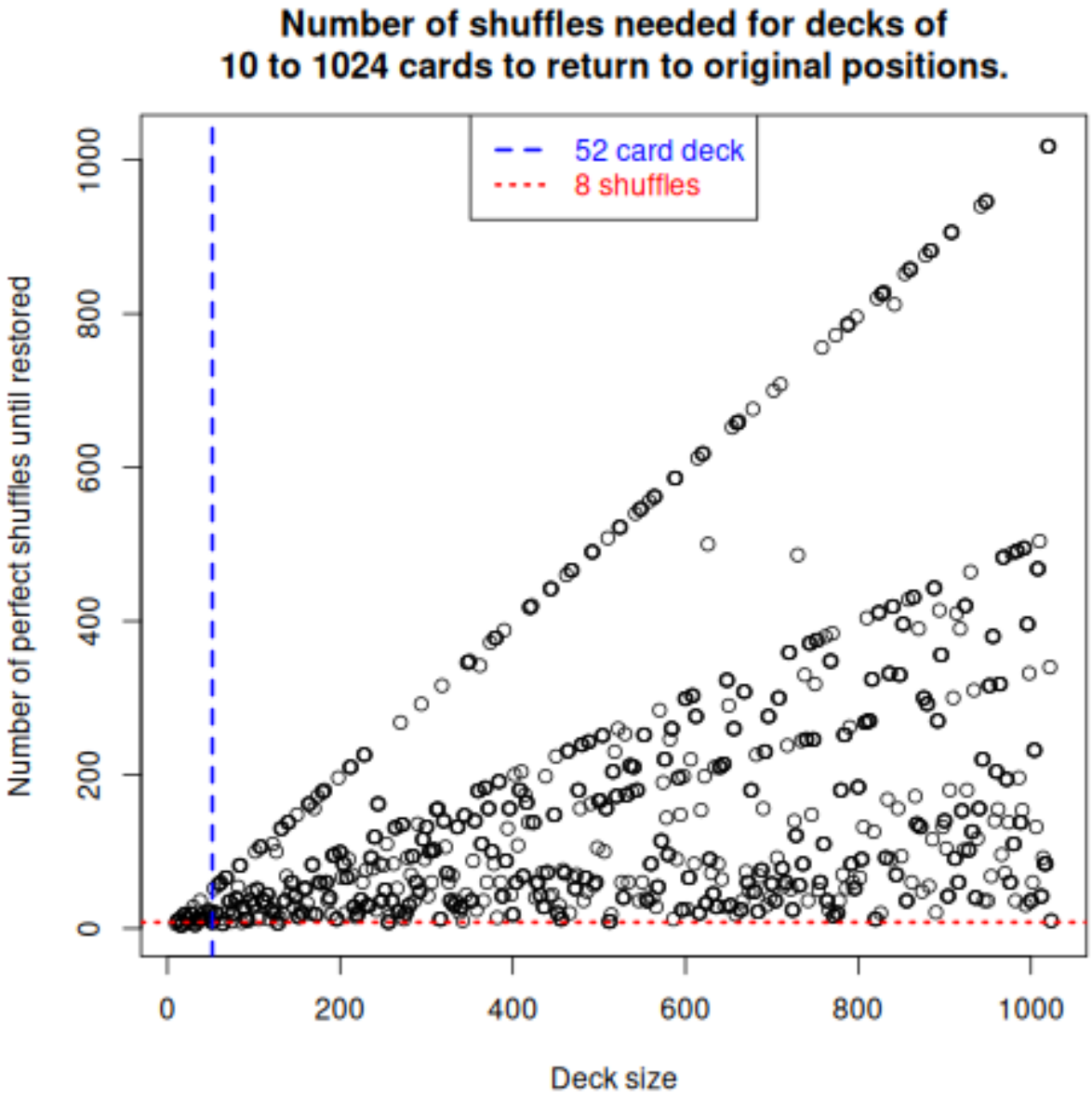

Figure 4: Number of perfect shuffles per deck size to return to original state. Deck sizes were varied from 10 to 1,024 to determine the number of shuffles required. In all cases, the deck could be returned to its original state, but the number varied considerably. There does not appear to be a pattern in the number of shuffles.

Figure 5: Assumed directory and file structure. If the relationship between `shuffle05.R` and the other directories/files where names are case sensitive.